Indian Statistical Institute, Bangalore

B. Math (Hons.) First Year

Second Semester - Analysis II

Mid-Semester Exam Date: 23rd February 2026

Maximum marks: 30 Duration: 2 hours

Answer any six and each question carries 5 marks

1. Prove that any continuous function on $[a, b]$ is Riemann-integrable.
2. Prove that $f \in \mathcal{R}[a, b]$ if and only if $f \in \mathcal{R}[a, x]$ and $f \in \mathcal{R}[x, b]$ for any $x \in [a, b]$.
3. If $f, g \in \mathcal{R}[a, b]$, prove that $f + g \in \mathcal{R}[a, b]$.
4. Let $f \geq 0$ be a decreasing continuously differentiable function on $[0, \infty)$ and $\int_0^\infty f(x)dx$ converge. Prove that $\lim_{x\to\infty} xf(x) = 0$ and $\int_0^\infty xf'(x)dx$ converges.
5. Determine all continuous $f\colon \mathbb{R} \to \mathbb{R}$ such that $f(x) = \int_0^1 f(x+t)dt$ for all $x \in \mathbb{R}$ and f attains its maximum on $\mathbb{R}$.
6. Let (f_n) be a sequence of differentiable functions on $[a, b]$. Assume that (f_n') converges uniformly on $[a, b]$ and $(f_n(x_0))$ converges for some $x_0 \in [a, b]$. Show that (f_n) converges uniformly to a function f on $[a, b]$ and $f_n' \to f'$.
7. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a uniformly continuous function. If $f_n(x) = f(x + 1/n)$ for all $x \in \mathbb{R}$, prove that (f_n) converges uniformly to f on $\mathbb{R}$. Is the result true if uniform continuity is replaced by continuity. Justify your answer.
8. Let (f_n) and (g_n) be sequences of bounded functions that converge uniformly. Prove that $(f_n g_n)$ converges uniformly.